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Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **20** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a power or product	**M1**	Ignoring the 3 or the 5 is not a misread.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 + (1-x)\ln 3$	**A1**	Condone invisible brackets if they are used correctly later.
	Solve for x	**M1**	Get as far as $x = \ldots$ Condone minor slips in the processing e.g. sign errors and losing a term that had been there, but award M0 for a fundamental error e.g. $3x\ln 2 + x\ln 3 = 3x\ln 6$ or ignoring the 3 or the 5 completely. Condone working in decimals.
	Obtain **final** answer $x = \dfrac{\ln 30}{\ln 24}$	**A1**	Do not ISW
	Alternative method for question 1		
	Use laws of indices to split at least one exponential term	**M1**	e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with 8^x and/or 3^x.
	Obtain $24^x = 30$	**A1**	Or equivalent e.g. $3^x 8^x = 30$ not for $3^x 2^{3x} = 30$ (need two factors with the same index).
	Solve for x	**M1**	Get as far as $x = \ldots$
	Obtain **final** answer $x = \dfrac{\ln 30}{\ln 24}$	**A1**	Do not ISW
		4	

Question	Answer	Marks	Guidance
2(a)	Substitute $x = -\frac{3}{2}$ and equate result to zero	**M1**	Or divide by $2x+3$ and set constant remainder equal to zero. Or state $\left(2x^3 - x^2 + a\right) = (2x+3)\left(x^2 + px + q\right)$, compare coefficients and solve for p or q.
	Obtain $a = 9$	**A1**	
		2	

Question	Answer	Marks	Guidance
2(b)	Commence division by $(2x+3)$ reaching a partial quotient $x^2 + kx$	*M1	The M1 is earned if inspection reaches an unknown factor: $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B.
	Obtain factorisation $(2x+3)(x^2 - 2x + 3)$	A1	Allow if the correct quotient seen. Correct factors seen in **(a)** and quoted or used here scores M1A1.
	Show that $x^2 - 2x + 3$ is always positive, or $2x^3 - x^2 + 9$ only intersects the x-axis once	DM1	Must use their quadratic factor. SC If M0, allow B1 if state $x < -\frac{3}{2}$ and no error seen
	State **final** answer $x < -\frac{3}{2}$ from correct work	A1	
		4	

Question	Answer	Marks	Guidance
3	Use correct product rule on given expression	*M1	
	Obtain correct derivative in any form	A1	e.g. $\cos x \sin 2x + 2\sin x \cos 2x$
	Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$	*M1	
	Equate derivative to zero and obtain an equation in one trig variable	DM1	dependent on the 2 previous M Marks.
	Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$	A1	OE
	Solve and obtain $x = 0.955$	A1	3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of y.
	Alternative method for the first three marks		
	Use correct double angle formula to obtain $y = 2\cos x - 2\cos^3 x$	*M1	or $y = 2\sin^2 x \cos x$
	Use chain rule and / or product rule	*M1	
	Obtain derivative $y' = -2\sin x + 6\sin x \cos^2 x$	A1	$y' = -2\sin^3 x + 4\sin x \cos^2 x$
	Alternative method for the second and third M marks		
	Equate derivative to zero and obtain an equation in $\tan x$ and $\tan 2x$	*M1	
	Use correct double angle formula to obtain an equation in $\tan x$	DM1	
		6	

Question	Answer	Marks	Guidance
4(a)	State $R = \sqrt{17}$	**B1**	Allow if working from an incorrect expansion but not from decimals.
	Use correct trig formulae to find α (Correct expansion and correct expression for trig ratio for α)	**M1**	NB: $\cos\alpha = 4$ and $\sin\alpha = 1$ scores M0A0. M0 for incorrect expansion of $\cos(x-\alpha)$ M1 for correct expression for trig ratio for α and no errors seen.
	Obtain $\alpha = 14.04°$	**A1**	2 d.p. required Allow M1A1 for correct answer with no working shown. Correct answer from incorrect working (e.g. $\tan^{-1}\left(-\frac{1}{4}\right)$) is awarded M0A0. $180° - \tan^{-1}\left(-\frac{1}{4}\right)$ is awarded M1
		3	
4(b)	Evaluate $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ to at least 1 d.p. (43.31 38…°)	**B1 FT**	FT *their R*. Accept awrt $43.3°$ or awrt $316.7°$ Can be implied by subsequent working.
	Use correct method to find a value of *x* in the interval	**M1**	Must be working with 2*x* and *their* α .
	Obtain answer, e.g. 14.6 °	**A1**	Accept overspecified answers but they need to be correct. (14.6388…. and 151.3249…).
	Use a correct method to find a second answer in the interval	**M1**	Must be working with 2*x*, *their* α and $360° -$ *their* 43.3 .
	Obtain second answer in the interval, e.g. 151.3 ° , and no other in the interval	**A1**	Ignore answers outside the given interval. Treat answers in radians (0.255… and 2.64…) as a misread.
		5	

Question	Answer	Marks	Guidance
5(a)	Use quadratic formula, or completing the square $\left((z-3\text{i})^2-3=0\right)$ and use $\text{i}^2=-1$ to find a root	**M1**	
	Obtain a root, e.g. $\sqrt{3}+3\text{i}$	**A1**	Or exact 2 term equivalent e.g. $\frac{6\text{i}}{2}+\frac{\sqrt{12}}{2}$ ISW.
	Obtain the other root, e.g. $-\sqrt{3}+3\text{i}$	**A1**	Or exact 2 term equivalent ISW.
		3	
5(b)	Show points representing the roots correctly	**B1 FT**	2 roots consistent with *their* **(a)** and with no errors seen on the diagram. B0 if they only have one root or more than 2 roots Must match their scale and $1<\sqrt{3}<2$ Linear scales seen or implied. Need some indication of scale (numbers or dashes). Scales along an axis must be approximately consistent but scales may be different on the 2 axes.
		1	

Question	Answer	Marks	Guidance
5(c)	State modulus of either root is $2\sqrt{3}$, or simplified exact equivalent	B1 FT	ISW if converted to decimal . Ignore modulus of second root if seen. Follow their root(s) not on either axis (from **(a)** or **(b)**).
	Find the argument of one of their roots – get as far as $\tan^{-1}(\ldots)$	M1	SOI but must be correct for their root.
	Obtain correct arguments $\frac{1}{3}\pi$ and $\frac{2}{3}\pi$, or simplified exact equivalents	A1	Must obtain values. Allow degrees.
		3	
5(d)	Give a **complete justification** that the **correct triangle** is equilateral	B1	Check *their* diagram in **(b)**. Possible justifications: 3 equal sides, or all angles equal to $\frac{\pi}{3}$, or isosceles and an angle of $\frac{\pi}{3}$.
		1	

Question	Answer	Marks	Guidance
6(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form	**B1**	$\left(\overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k},\quad \overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{k}\right)$.
	Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$,	**M1**	or $\overrightarrow{BA}.\overrightarrow{CA}$ $(8 - 3 = 5)$. M0 for $\overrightarrow{AB}.\overrightarrow{CA}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of *their* moduli to obtain $\cos\theta$ or θ	**M1**	$\left(\dfrac{5}{\sqrt{9}\sqrt{25}}\right)$ Independent of the first M1.
	Obtain answer $\dfrac{1}{3}$	**A1**	ISW. Need to see a value for $\cos\theta$. Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1}\frac{1}{3}$ alone is not sufficient)
		4	

Question	Answer	Marks	Guidance
6(b)	Use correct method to find an **exact** value for the sine of angle *BAC* from *their* **(a)**	**M1**	$\left(\sqrt{1-\frac{1}{9}}\right)$
	Obtain answer $\frac{2}{3}\sqrt{2}$, or equivalent	**A1**	
	Use correct area formula to find the area of triangle *ABC* with *their* versions of relevant vectors	**M1**	$\left(\frac{1}{2}\sqrt{9}\sqrt{25}\times \textit{their}\ \sin\theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25}\times\sin\left(\cos^{-1}\frac{1}{3}\right)$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	Only ISW
	Alternative method 1 for question 6(b)		
	Use correct method to find the perpendicular distance from *A* to *BC* (or *B* to *AC* or *C* to *AB*)	**M1**	$\begin{pmatrix}2+2\lambda\\-2+2\lambda\\1-4\lambda\end{pmatrix}.\begin{pmatrix}2\\2\\-4\end{pmatrix}=0\Rightarrow\lambda=\frac{1}{6}$
	Obtain $\frac{1}{3}\sqrt{75}$	**A1**	$\left(\left\|\frac{7}{3}\mathbf{i}-\frac{5}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}\right\|\right)$
	Use correct area formula to find the area of triangle *ABC*	**M1**	$\left(\frac{1}{2}\times \textit{their}\sqrt{24}\times \textit{their}\,\frac{1}{3}\sqrt{75}\right)$ The length they use for *their* base must be found correctly.
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	

Question	Answer	Marks	Guidance
6(b)	**Alternative method 2 for question 6(b)**		
	Correct method to find the semi-perimeter	**M1**	
	Obtain $4+\sqrt{6}$	**A1**	
	Correct application of Hero's (Heron's) formula	**M1**	$\sqrt{\left(4+\sqrt{6}\right)\left(1+\sqrt{6}\right)\left(-1+\sqrt{6}\right)\left(4-\sqrt{6}\right)}$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given statement	**B1**	e.g. see $2\cot\theta \times -\operatorname{cosec}^2\theta$ in the working or express in terms of $\sin\theta$ and $\cos\theta$ and use quotient rule to obtain the given result. Solution must have θ present throughout and must reach the given answer.
		1	

Question	Answer	Marks	Guidance
7(b)	Separate variables correctly Check for relevant working in **(a)**	**B1**	$\int x\,dx = \int \frac{\tan^2\theta}{\sin^2\theta} - \frac{2\cot\theta}{\sin^2\theta}\,d\theta$ Condone incorrect notation e.g. missing d*x*. Need either the integral sign or the d*x*, d*θ*.
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain terms $\tan\theta + \cot^2\theta$	**B1 + B1**	Alternative: $\int \frac{2\cot\theta}{\sin^2\theta}\,d\theta = \int \frac{2\cos\theta}{\sin^3\theta}\,d\theta = -\frac{1}{\sin^2\theta}(+C)$
	Form an equation for the constant of integration, or use limits $x = 2,\ \theta = \frac{1}{4}\pi$, in a solution with **at least two correctly obtained terms** of the form ax^2, $b\tan\theta$ and $c\cot^2\theta$, where $abc \neq 0$	**M1**	Need to have 3 terms. Constant of correct form.
	State correct solution in any form, e.g. $\frac{1}{2}x^2 = \tan\theta + \cot^2\theta$	**A1**	or $\frac{1}{2}x^2 = \tan\theta + \operatorname{cosec}^2\theta - 1$ If everything else is correct, allow a correct final answer to imply this A1.
	Substitute $\theta = \frac{1}{6}\pi$ and obtain answer $x = 2.67$	**A1**	2.6748… $\sqrt{\frac{18+2\sqrt{3}}{3}}$ If see a correctly rounded value ISW.
		7	

Question	Answer	Marks	Guidance
8(a)	State ($a =$) π^2	B1	Allow 32400, 180^2. Accept $(x =)\pi^2$.
		1	
8(b)	State or imply $\mathrm{d}x = 2u\ \mathrm{d}u$ or equivalent	B1	e.g. $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $\mathrm{d}u = \frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and $\mathrm{d}x$ throughout the integral	M1	
	Obtain $\int 2u \sin u\, \mathrm{d}u$	A1	Allow with missing du.
	Commence integration of $\int ku \sin u\, \mathrm{d}u$ by parts and reach $\mp ku \cos u \pm \int k \cos u\, \mathrm{d}u$	*M1	
	Obtain integral $-ku \cos u + k \sin u$	A1	
	Substitute limits $u = 0$ and $u = \sqrt{their\ a}$, $a \neq 0$, a in radians or $x = 0$ and *their a* in the complete integral	DM1	$-2\pi \cos \pi + 2 \sin \pi (+0 - 2 \sin 0)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	A1	
		7	

Question	Answer	Marks	Guidance
9(a)	State or imply angle $AOC = \pi - 2\theta$	**B1**	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	**M1**	$\frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2\sin(\pi - 2\theta)$ or $\frac{1}{2}\pi r^2 - \left[\frac{1}{2}r^2(2\theta) + \frac{1}{2}r^2\sin(\pi - 2\theta)\right]$ M0 if subtraction the wrong way round.
	Equate to $\dfrac{1}{6}\pi r^2$ and obtain a correct equation in any form	**A1**	e.g. $\frac{1}{6}\pi r^2 = \frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2\sin(\pi - 2\theta)$.
	Obtain $\theta = \frac{1}{3}(\pi - 1.5\sin 2\theta)$ correctly	**A1**	AG Condone if state / imply $\sin(\pi - 2\theta) = \sin 2\theta$.
		4	
9(b)	Evaluate a relevant expression or pair of expressions at $\theta = 0.5$ and $\theta = 0.7$	**M1**	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x = \mathrm{f}(x)$ embedded values are not sufficient e.g. f(0.5) …. is accepted but $\frac{1}{3}(\pi - 1.5\sin 2 \times 0.5) = ...$ is not.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $0.5 < 0.626, 0.7 > 0.554$ or $0.126 > 0, -0.146 < 0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	

Question	Answer	Marks	Guidance
9(c)	Use the iterative process $\theta_{n+1} = \frac{1}{3}(\pi - 1.5\sin 2\theta_n)$ correctly at least once	M1	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	A1	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	A1	e.g. $0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, 0.58599, 0.58644$ $0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636$ $0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636$ Allow working to more than 5 dp, but not less.
		3	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1+x}+\frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	**A1**	**SC**: A maximum of M1A1 is available for obtaining $A=2$ after scoring B0.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
10(b)	Integrate and obtain term $2\ln(1+x)$	**B1 FT**	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	***M1**	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	**A1 FT**	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab \neq 0$	**DM1**	Ignore working relating to $C \neq 0$. $\left(2\ln 5-2\ln 1-\frac{1}{2}\ln 18+\frac{1}{2}\ln 2\right)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	**A1**	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	